



Calculations Policy

March 2015

*'The National Curriculum for mathematics aims to ensure that all pupils become **fluent** in the fundamentals of mathematics... **reason mathematically**.... and can **solve problems**....'*

'Mathematics is an interconnected subject...Pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.'

Early Learning Goal

Mathematics - Number- addition and subtraction

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

MilestoneOne:

Using number facts

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

Y1 Represent and use number bonds and related subtraction facts within 20

Y2 Recall and use addition and subtraction facts to 20 fluently, and derive and use related number facts up to 100.

Complexity

Apply increasing knowledge of mental and written methods.

Y1 Add and subtract one-digit and two digit numbers to 20.

Y2 (non-stat. guidance) Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.

Apply their increasing knowledge of mental and written methods.

Milestone Two:

Using number facts

Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction.

Complexity

Solve two-step addition and subtraction problems in contexts, deciding which operations and methods to use and why.

Y3 Add and subtract numbers mentally, including: a three digit number and ones, a three digit number and tens, a three digit number and hundreds.

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Solve problems including missing number problems using number facts, place value and more complex addition and subtraction.

Y4 Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.

Solve addition and subtraction two-step problems in contexts, deciding which operations to use and why.

Milestone Three:

Methods

Add and subtract whole numbers with more than 4 digits, including using formal written methods. (columnar addition and subtraction)

Add and subtract numbers mentally with increasingly large numbers.

Complexity

Solve multi-step addition and subtraction problems in contexts, deciding which operations and methods to use and why.

Y5 Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction).

Solve addition and subtraction multi-step problems in contexts, deciding which operations to use and why.

Y6 Undertake mental calculations with increasingly large numbers and more complex calculations.

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

PROGRESSION THROUGH CALCULATIONS FOR ADDITION

MENTAL CALCULATIONS

Mental recall of number bonds

$$6 + 4 = 10$$

$$\square + 3 = 10$$

$$25 + 75 = 100$$

$$19 + \square = 20$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)}$$

$$460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43$$

$$458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Reception and Key Stage One

Written Methods

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures.

Early Learning Goal

Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Milestone One:

Methods

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- One-digit and two-digit numbers to 20, including zero.
- A two-digit number and ones.

- A two-digit number and tens.
- Two two-digit numbers.
- Adding three one-digit numbers.

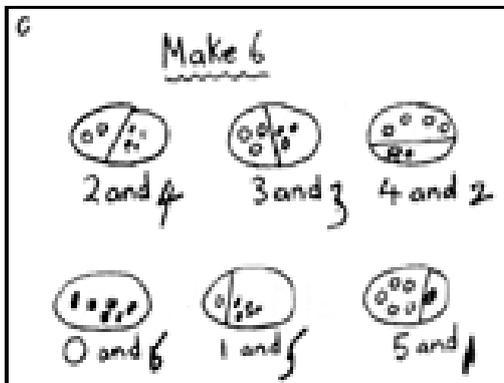
Milestone One:

Using number facts

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

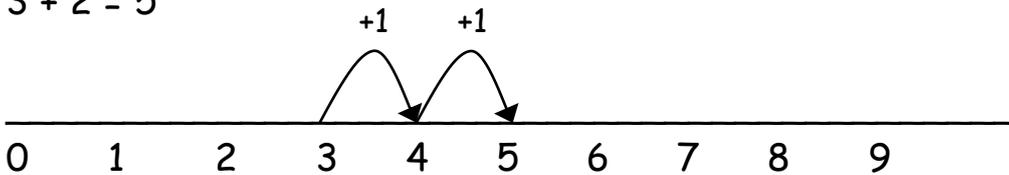
Y1 Represent and use number bonds and related subtraction facts within 20

Y2 Recall and use addition and subtraction facts to 20 fluently, and derive and use related number facts up to 100.



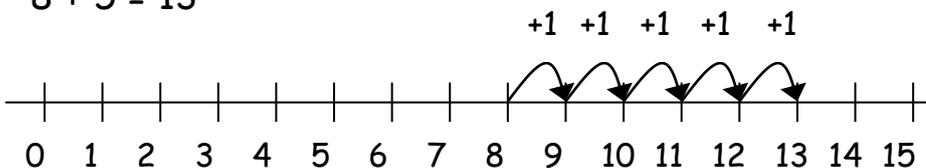
They use numberlines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.

$$3 + 2 = 5$$

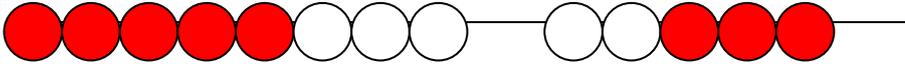


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

$$8 + 5 = 13$$



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

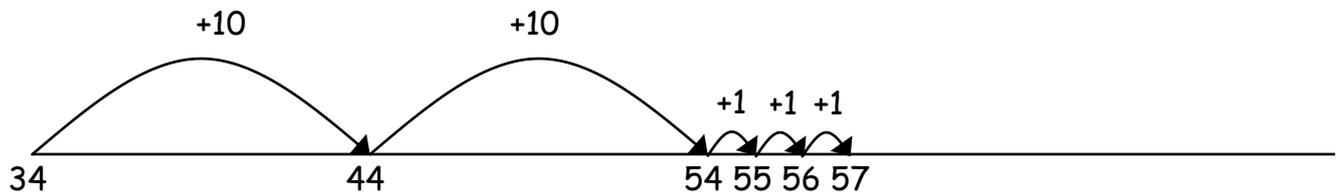


Y2

Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

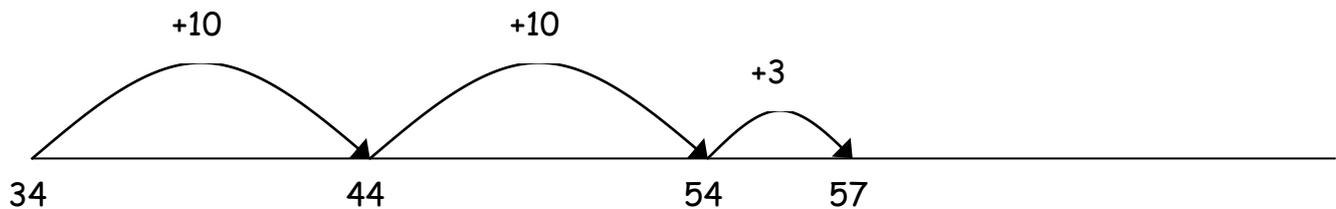
- ✓ First counting on in tens and ones.

$$34 + 23 = 57$$



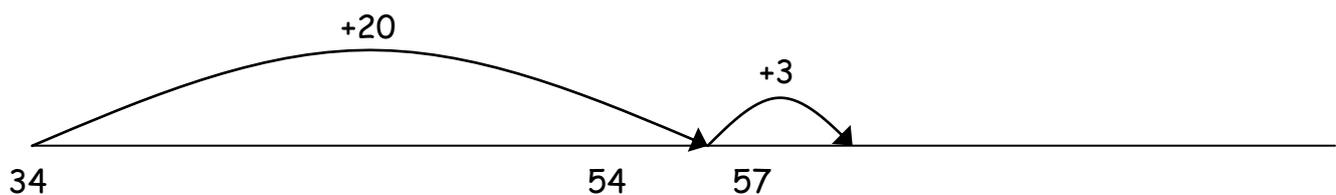
- ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact $4 + 3 = 7$).

$$34 + 23 = 57$$



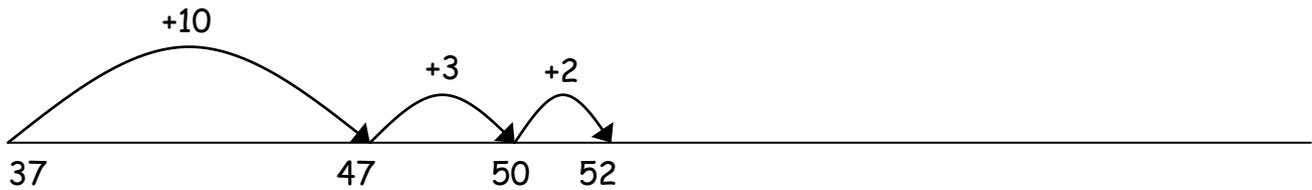
- ✓ Followed by adding the tens in one jump and the units in one jump.

$$34 + 23 = 57$$



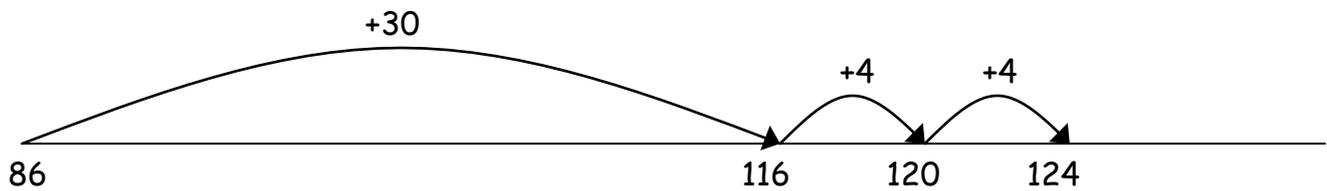
- ✓ Bridging through ten can help children become more efficient.

$$37+15=52$$



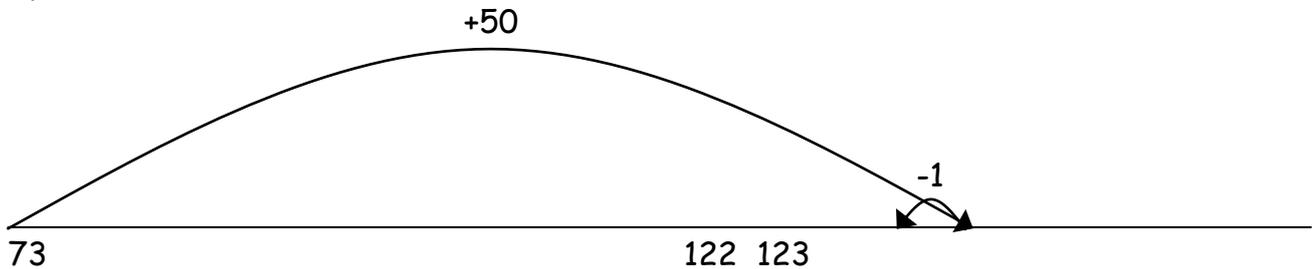
✓ Count on from the largest number irrespective of the order of the calculation.

$$38+86 = 124$$



✓ Compensation

$$49+ 73= 122$$



Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Y3

Assessment criteria for mathematics

Milestone Two

Mental Maths

Methods

- Add and subtract numbers mentally, including:

- A three-digit number and ones.
- A three-digit number and tens.
- A three-digit number and hundreds

Year 3 (Non-stat guidance)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two digit numbers, the answers could exceed 100.

Written Methods

Pupils use their understanding of place value and partitioning and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent.

Assessment criteria for mathematics

Milestone Two

Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.

Adding the least significant digits first in preparation for 'carrying'.

$$\begin{array}{r}
 67 \\
 + 24 \\
 \hline
 11 \text{ (} 7 + 4 \text{)} \\
 80 \text{ (} 60 + 20 \text{)} \\
 \hline
 91
 \end{array}$$

$$\begin{array}{r}
 267 \\
 + 85 \\
 \hline
 12 \text{ (} 7 + 5 \text{)} \\
 140 \text{ (} 60 + 80 \text{)} \\
 \hline
 200 \\
 \hline
 352
 \end{array}$$

From this, children will begin to carry below the line.

$$\begin{array}{r}
 625 \\
 + 48 \\
 \hline
 673 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 783 \\
 + 42 \\
 \hline
 825 \\
 \hline
 1
 \end{array}$$

$$\begin{array}{r}
 367 \\
 + 85 \\
 \hline
 452 \\
 \hline
 11
 \end{array}$$

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;*
- ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*

Mathematics Appendix 1: Examples of formal written methods for addition and subtraction.

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline 1 \quad 1 \end{array}$$

Answer: 1431

874 - 523 becomes

$$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$$

Answer: 351

932 - 457 becomes

$$\begin{array}{r} 8 \quad 12 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

932 - 457 becomes

$$\begin{array}{r} 1 \quad 1 \\ 932 \\ - 457 \\ \hline 5 \quad 6 \\ \hline 475 \end{array}$$

Answer: 475

Y4

Year 4 (Non-stat guidance)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency

Children should extend the carrying method to numbers with up to four digits.

$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline 1 \quad 1 \end{array}$$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ \hline 1 \quad 1 \quad 1 \end{array}$$

Using similar methods, children will:

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.*

Y5

Milestone Three

Methods

Add and subtract whole numbers with more than 4 digits, including using formal written methods. (columnar addition and subtraction)

- Add and subtract numbers mentally with increasingly large numbers.

Children should extend the carrying method to numbers with any number of digits.

$$\begin{array}{r} 7648 \\ + 1486 \\ \hline 9134 \\ \hline \end{array}$$

1 11

$$\begin{array}{r} 6584 \\ + 5848 \\ \hline 12432 \\ \hline \end{array}$$

1 11

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ 3 \\ + 4681 \\ \hline 11944 \\ \hline \end{array}$$

1 2 1

Using similar methods, children will

- ✓ *add several numbers with different numbers of digits;*
- ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;*
- ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$.*

By the end of Year 6, pupils should be fluent in written methods for all four operations including long multiplication and division, and in working with fractions, decimals and percentages.

. + - + - + - + - + - +

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Children should be encouraged to estimate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.

PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

MENTAL CALCULATIONS

Mental recall of addition and subtraction facts

$10 - 6 = 4$

$17 - \square = 11$

$20 - 17 = 3$

$10 - \square = 2$

Find a small difference by counting up

$82 - 79 = 3$

Counting on or back in repeated steps of 1, 10, 100, 1000

$86 - 52 = 34$ (by counting back in tens and then in ones)

$460 - 300 = 160$ (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$$24 - 19 = 24 - 20 + 1 = 5$$

$$458 - 71 = 458 - 70 - 1 = 387$$

Use the relationship between addition and subtraction

$$36 + 19 = 55$$

$$19 + 36 = 55$$

$$55 - 19 = 36$$

$$55 - 36 = 19$$

Reception and Key Stage One

Early Learning Goal

Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.

Milestone One:

Methods

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- One-digit and two-digit numbers to 20, including zero.
- A two-digit number and ones.
- A two-digit number and tens.
- Two two-digit numbers.
- Adding three one-digit numbers.

Milestone One:

Using number facts

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

Y1 Represent and use number bonds and related subtraction facts within 20

Y2 Recall and use addition and subtraction facts to 20 fluently, and derive and use related number facts up to 100.

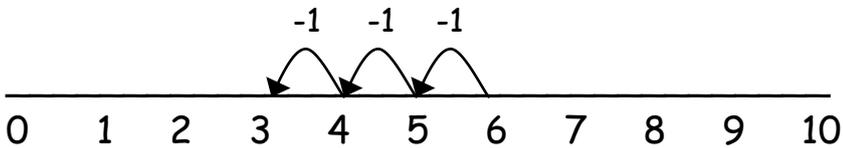
Written Methods

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures.

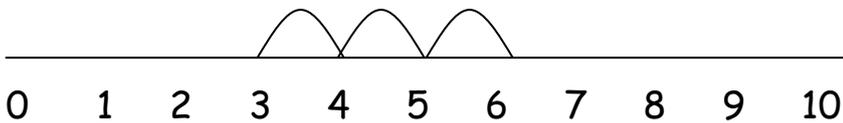


They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.

$$6 - 3 = 3$$



The numberline should also be used to show that $6 - 3$ means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.

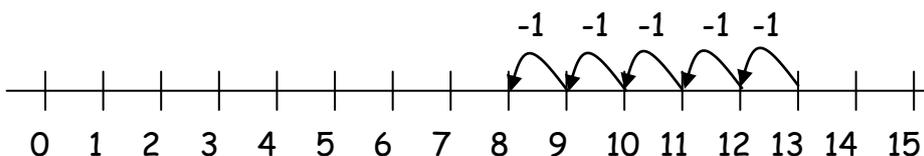


Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.

Y1

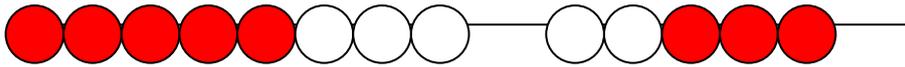
Add or subtract one-digit and two digit numbers to 20 including zero

$$13 - 5 = 8$$



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



Y2

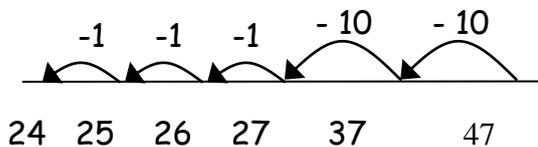
Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.

Add and subtract numbers using concrete objects, pictorial representations and mentally including:

- a two digit number and ones
- a two digit number and tens
- twodigit numbers
- adding three one digit numbers

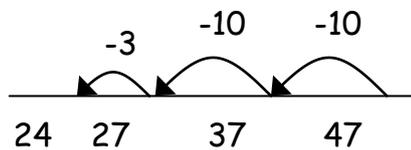
✓ First counting back in tens and ones.

$$47 - 23 = 24$$



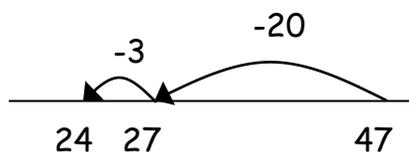
✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



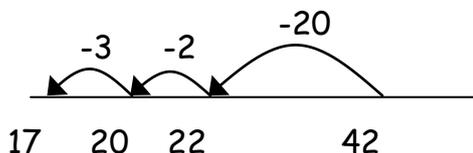
✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$

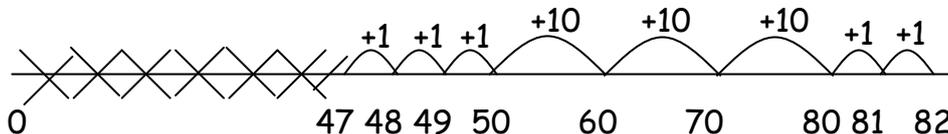


If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$$82 - 47$$



Help children to become more efficient with counting on by:

- ✓ Subtracting the units in one jump;
- ✓ Subtracting the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

Children will continue to use empty number lines with increasingly large numbers.

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Partitioning and decomposition

This process should be demonstrated using arrow cards to show the partitioning and base 10 materials to show the decomposition of the number.

$$\begin{array}{r} 89 \\ - 57 \\ \hline \end{array} = \begin{array}{r} 80 \\ + 50 \\ \hline \end{array} + 9 + 7$$

$$30 + 2 = 32$$

Initially, the children will be taught using examples that do not need the children to exchange.

Y3

Partitioning and Column Method

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array} =$$

$$\text{Step 1} \quad \begin{array}{r} 700 + 50 + 4 \\ - 80 + 6 \\ \hline \end{array}$$

$$\text{Step 2} \quad \begin{array}{r} 700 + 40 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from T to U})$$

$$\text{Step 3} \quad \begin{array}{r} 600 + 140 + 14 \\ - \quad \quad 80 + 6 \\ \hline \end{array} \quad (\text{adjust from H to T})$$

$$600 + 60 + 8 = 668$$

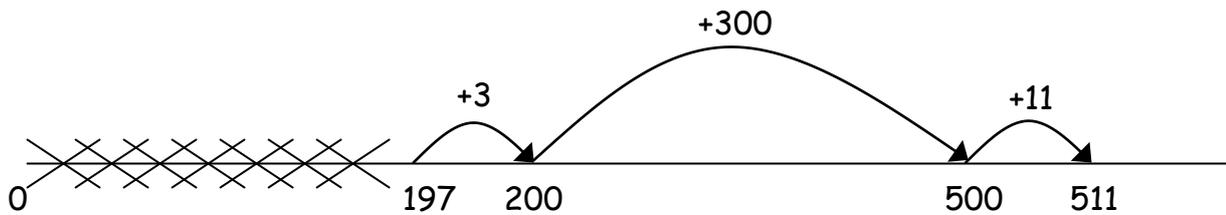
This would be recorded by the children as

$$\begin{array}{r} 600 \quad 140 \\ \quad \quad \quad \cancel{700} + \cancel{50} + 14 \\ \quad \quad \quad - \quad \quad \underline{80 + 6} \\ \quad \quad \quad 600 + 60 + 8 = 668 \end{array}$$

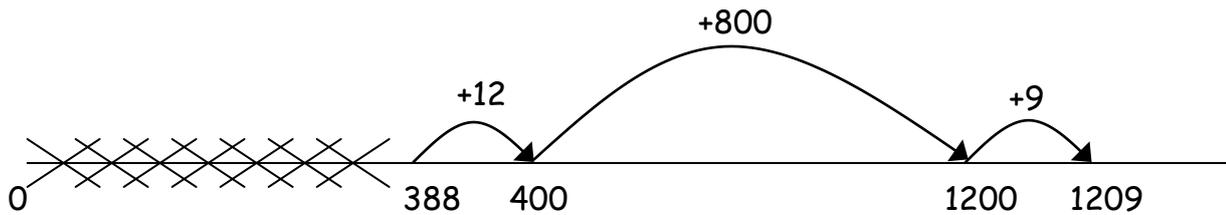
Alternatively, children can set the amounts to whole numbers, i.e. 895 - 438 and convert to pounds after the calculation.

Counting on

$$511 - 197 = 314$$



$$1209 - 388 = 821$$



Mathematics Appendix 1: Examples of formal written methods for addition and subtraction.

789 + 642 becomes

$$\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ \hline 11 \end{array}$$

Answer: 1431

874 - 523 becomes

$$\begin{array}{r} 874 \\ - 523 \\ \hline 351 \end{array}$$

Answer: 351

932 - 457 becomes

$$\begin{array}{r} 8 \quad 12 \quad 1 \\ 932 \\ - 457 \\ \hline 475 \end{array}$$

Answer: 475

932 - 457 becomes

$$\begin{array}{r} 1 \quad 1 \\ 932 \\ - 457 \\ \hline 5 \quad 6 \\ 475 \end{array}$$

Answer: 475

Y4

Column Method

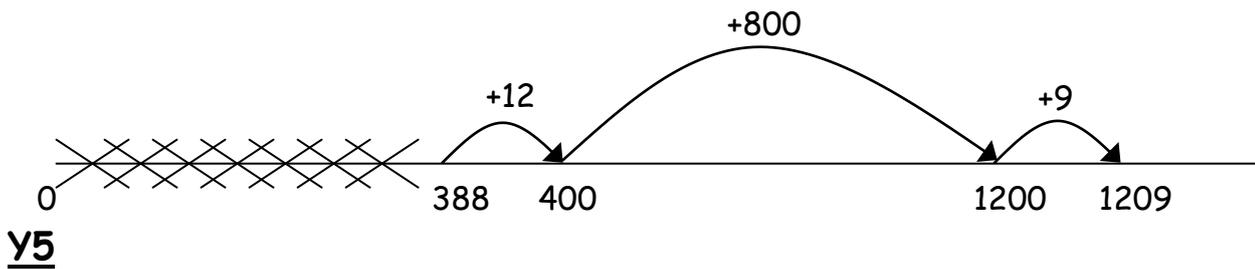
614 1

$$\begin{array}{r} 614 \quad 1 \\ - 286 \\ \hline 468 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other.

$$1209 - 388 = 821$$



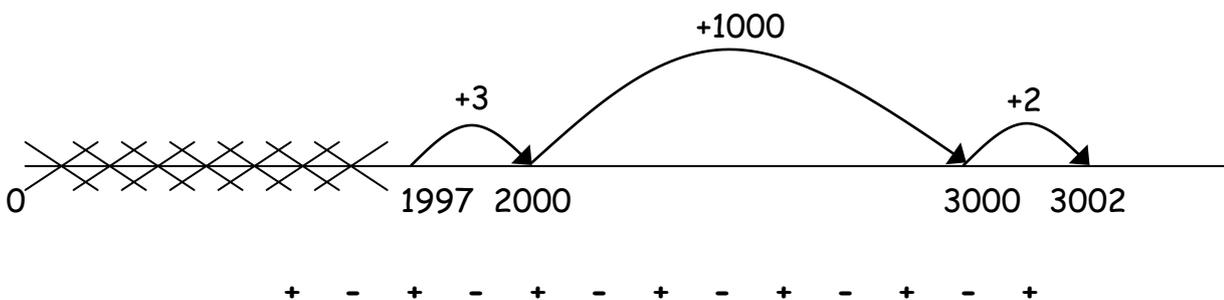
$$\begin{array}{r} ^5 ^{13} ^1 \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}$$

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other.

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

$$3002 - 1997 = 1005$$



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should be encouraged to approximate their answers before calculating. Children should be encouraged to check their answers after calculation using an appropriate strategy.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

Early Learning Goal

Mathematics - Number- Multiplication and division

They solve problems, including doubling, halving and sharing.

Milestone One

Using multiplication and division facts

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables.
- Recognise odd and even numbers.
- Use multiplication and division facts to solve problems.

Y1 Solve one-step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher that involve arrays.

Y2 (non-stat guidance) They begin to use other multiplication tables and recall multiplication facts, including related division facts to perform written and mental calculations.

Milestone Two

Using multiplication and division facts

- Recall multiplication and division facts for multiplication tables up to 12×12 .

Y3 Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers using mental methods and progressing to formal written methods.

Y4 Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Milestone Three

Using multiplication and division facts

- Identify common factors, common multiples and prime numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3).
- Solve problems involving multiplication and division including using knowledge of factors and multiples, squares and cubes.

Y5 Extend use of formal written methods: Short multiplication and short division.

Y6 All written methods including long division

MENTAL CALCULATIONS

Doubling and halving

Applying the knowledge of doubles and halves to known facts.

e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table
 4 times table
 5 times table
 8 times table
 10 times table

Year 4 Recall quickly all multiplication facts up to 12×12

Years 5 & 6 Apply all multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$13 \times 11 = (13 \times 10) + (13 \times 1)$
 $= 130 + 13$
 $= 143$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$23 \times 4 = (20 \times 4) + (3 \times 4)$
 $= 80 + 12$
 $= 102$

Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

Reception and Key Stage One

Early Learning Goal

Mathematics - Number- Multiplication and division

They solve problems, including doubling, halving and sharing.

Y1

Pupils should be taught to:

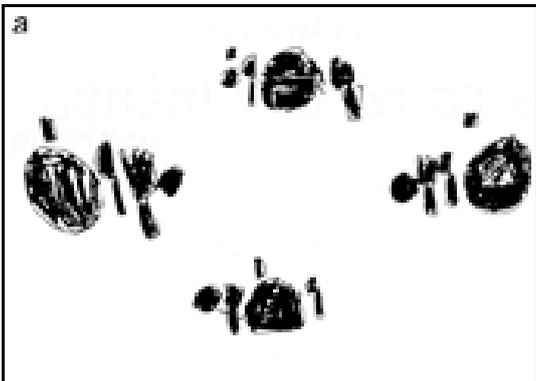
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher.

Y2

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts including problems in context.

Written methods

Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.



Milestone One

Method

- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals ($=$) signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division using mental methods.

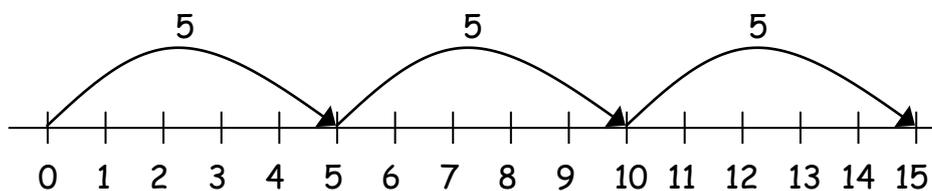
Children will develop their understanding of multiplication and use jottings to support calculation:

✓ **Repeated addition**

3 times 5 is $5+5+5=15$ or 3 lots of 5 or 5×3

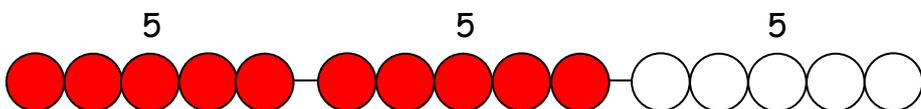
Repeated addition can be shown easily on a number line:

$$5 \times 3 = 5 + 5 + 5$$



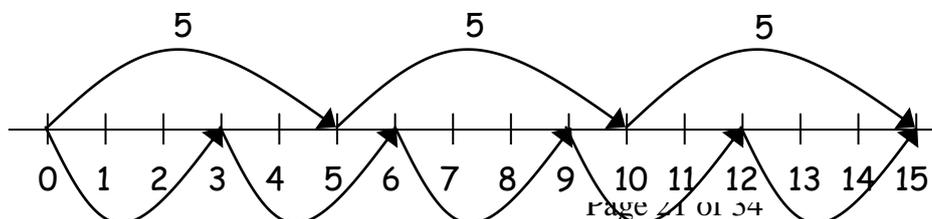
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



✓ **Commutativity**

Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \circ = 32$$

Y3

Number - multiplication and division

Pupils should be taught to:

- *recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables*
- *write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods*
- *solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects*

Milestone Two

Method

- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.
- Recognise and use factor pairs and commutativity in mental calculations.

TU x U

(Short multiplication - multiplication by a single digit)

$$23 \times 8$$

Start with Grid Method

23×8 is approximately $25 \times 8 = 200$

Then progressing to using formal Written Method

$$\begin{array}{r}
 \times \quad 20 \quad 3 \\
 8 \quad \boxed{160} \quad \boxed{24} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 160 \\
 + \quad 24 \\
 \hline
 184
 \end{array}$$

$$\begin{array}{r}
 24 \\
 \times 6 \\
 \hline
 \end{array}$$

Then progressing to using formal written method as shown:

Mathematics Appendix 1: Examples of formal written methods for short multiplication.

Y3

24×6 becomes

$$\begin{array}{r}
 24 \\
 \times 6 \\
 \hline
 144 \\
 2
 \end{array}$$

Answer: 144

Y4

342×7 becomes

$$\begin{array}{r}
 342 \\
 \times 7 \\
 \hline
 2394 \\
 21
 \end{array}$$

Answer: 2394

2741×6 becomes

$$\begin{array}{r}
 2741 \\
 \times 6 \\
 \hline
 16446 \\
 42
 \end{array}$$

Answer: 16 446

Y5

Multiply numbers up to 4 digits by a one or two-digit number using a formal written method including long multiplication for two digit numbers.

Y6

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Milestone Three

Method

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.
- Perform mental calculations, including with mixed operations and large numbers.

Mathematics Appendix 1: Examples of formal written methods for long multiplication.

24 × 16 becomes

$$\begin{array}{r} ^2 \\ 24 \\ \times 16 \\ \hline 240 \\ 144 \\ \hline 384 \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} ^1 ^2 \\ 124 \\ \times 26 \\ \hline 2480 \\ 744 \\ \hline 3224 \\ \hline ^1 ^1 \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} ^1 ^2 \\ 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ \hline ^1 ^1 \end{array}$$

Answer: 3224

Mathematics Appendix 2: Examples of formal written methods for chunking with larger numbers.

$$124 \times 65$$

X	100	20	4
60	6000	1200	240
5	500	100	20

$$\begin{array}{r} 6000 \\ 1200 \\ 240 \\ 500 \\ 100 \\ 20 \\ \hline 8060 \\ \hline 1 \end{array}$$

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should be encouraged to approximate their answers before calculating.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

PROGRESSION THROUGH CALCULATIONS FOR DIVISION

MENTAL CALCULATIONS

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught everyday from Y2 onwards, either as part of the mental oral starter or other times as appropriate within the day.

Year 2 2 times table
 5 times table
 10 times table

Year 3 2 times table
 3 times table

4 times table
5 times table
8 times table
10 times table

Year 4 Recall quickly division facts for all tables up to 12×12

Year 5 & 6 Apply all multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts.

e.g. If I know $3 \times 7 = 21$, what else do I know?

$30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\,000$, $0.3 \times 7 = 2.1$ etc

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$378 \div 21$ $378 \div 3 = 126$ $378 \div 21 = 18$
 $126 \div 7 = 18$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?

Reception and Key Stage One

Early Learning Goal

Mathematics - Number- Multiplication and division

They solve problems, including doubling, halving and sharing.

Y1

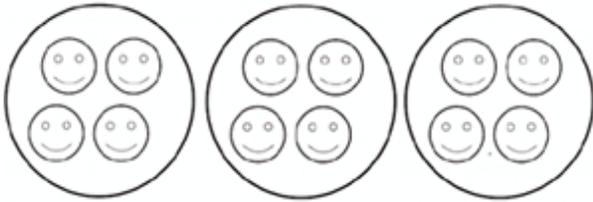
Pupils should be taught to:

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with support of the teacher.

Y2

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts including problems in context.

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

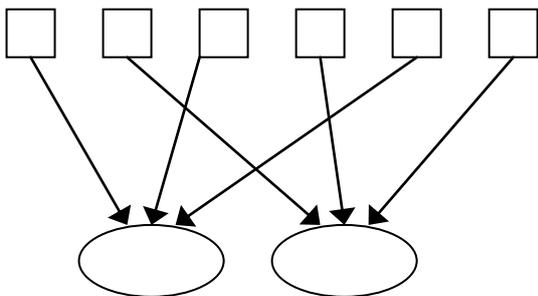


Children will develop their understanding of division and use jottings to support calculation.

Y2 (non-stat guidance) They begin to use other multiplication tables and recall multiplication facts, including related division facts to perform written and mental calculations.

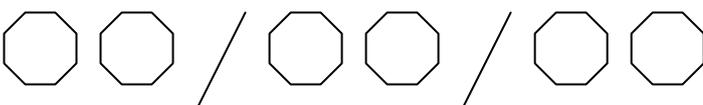
✓ **Sharing equally**

6 sweets shared between 2 people, how many do they each get?



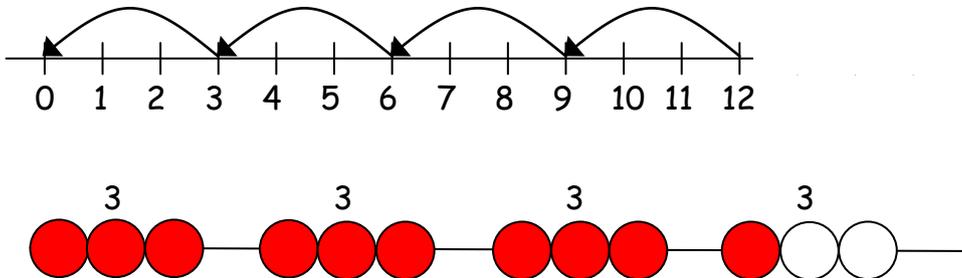
✓ **Grouping or repeated subtraction**

There are 6 sweets, how many people can have 2 sweets each?



- ✓ Repeated subtraction using a number line or bead bar

$$12 \div 3 = 4$$



The bead bar will help children with interpreting division calculations such as $10 \div 5$ as 'how many 5s make 10?'

- ✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$\square \div 2 = 4 \qquad 20 \div \triangle = 4 \qquad \square \div \triangle = 4$$

Y3

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

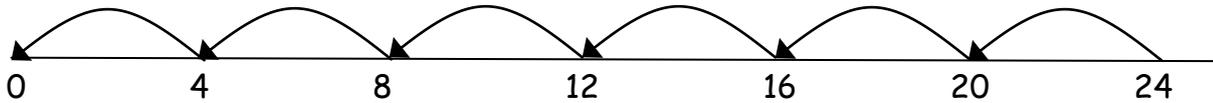
Notes and guidance (non-statutory)

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

- ✓ Repeated subtraction using a number line

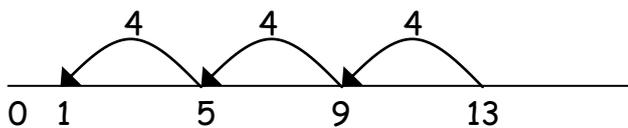
Children will use an empty number line to support their mental calculation.

$$24 \div 4 = 6$$



Children should also move onto calculations involving remainders.

$$13 \div 4 = 3 \text{ r } 1$$



✓ Using symbols to stand for unknown numbers to complete equations using inverse operations

$$26 \div 2 = \square \quad 24 \div \triangle = 12 \quad \square \div 10 = 8$$

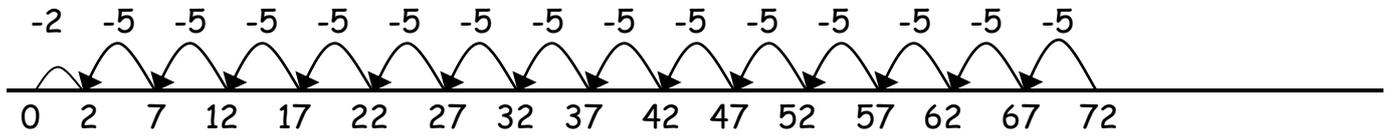
Y4

Pupils should be taught to:

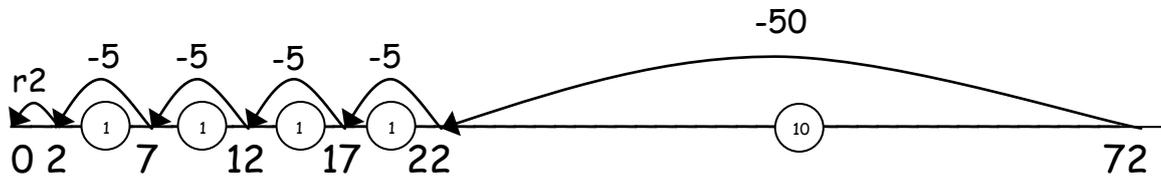
- *recall multiplication and division facts for multiplication tables up to 12×12*
- *use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers*
- *recognise and use factor pairs and commutativity in mental calculations*
- *multiply two-digit and three-digit numbers by a one-digit number using formal written layout*
- *solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects*

Children will develop their use of repeated subtraction supporting mental calculations to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s - numbers with which the children are more familiar.

$$72 \div 5$$



Moving onto:



Then onto a vertical method:

Division $TU \div U$

$$72 \div 3$$

$$\begin{array}{r}
 3 \overline{) 72} \\
 \underline{- 30} \\
 42 \\
 \underline{- 30} \quad 10x \\
 12 \\
 \underline{- 6} \quad 2x \\
 6 \\
 \underline{- 6} \quad 2x \\
 0
 \end{array}$$

Answer : 24

Leading to subtraction of other multiples.

$$96 \div 6$$

$$\begin{array}{r}
 6 \overline{) 96} \\
 \underline{- 60} \\
 36 \\
 \underline{- 36} \quad 6x \\
 0
 \end{array}$$

Answer : 16

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

Y4 (Non statutory - Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number)

Mathematics Appendix 1: Examples of formal written methods for short division

Y4

$98 \div 7$ becomes

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Answer: 14

$432 \div 5$ becomes

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{40} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Answer: 86 remainder 2

$496 \div 11$ becomes

$$\begin{array}{r} 45 \text{ r } 1 \\ 11 \overline{) 496} \\ \underline{44} \\ 56 \\ \underline{55} \\ 1 \end{array}$$

Answer: $45 \frac{1}{11}$

Y5

(Non statutory- Pupils practise and extend their uses of the formal written methods of short division.)

Pupils should be taught to:

- *divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context*
 - *multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000*
 - *solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes*
 - *solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign*
- solve problems involving multiplication and division, including scaling by fractions and problems involving simple rates.*

Y6

Pupils should be taught to:

- *divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context*
- *divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context*
- *perform mental calculations, including with mixed operations and large numbers*
- *use their knowledge of the order of operations to carry out calculations involving the 4 operations*
- *solve problems involving addition, subtraction, multiplication and division*
- *use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy*

Milestone 3

- *Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.*
- *Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.*

Mathematics Appendix 1: Examples of formal written methods for long division

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{300} \\
 132 \\
 \underline{120} \\
 12
 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r}
 28 \\
 15 \overline{) 432} \\
 \underline{300} \quad 15 \times 20 \\
 132 \\
 \underline{120} \quad 15 \times 8 \\
 12
 \end{array}$$

$$\frac{12}{15} = \frac{4}{5}$$

Answer: $28 \frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r}
 28.8 \\
 15 \overline{) 432.0} \\
 \underline{300} \quad \downarrow \\
 132 \\
 \underline{120} \quad \downarrow \\
 120 \\
 \underline{120} \\
 0
 \end{array}$$

Answer: 28.8

Appendix 2: Examples of written method for chunking by larger multiples

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{- 150} \quad 10 \times 15 \\
 282 \\
 \underline{- 150} \quad 10 \times 15 \\
 132 \\
 \underline{- 120} \quad 8 \times 15 \\
 12
 \end{array}$$

$$\begin{array}{r}
 28 \text{ r } 12 \\
 15 \overline{) 432} \\
 \underline{- 300} \quad 20 \times 15 \\
 132 \\
 \underline{- 120} \quad 8 \times 15 \\
 12
 \end{array}$$

By the end of Year 6, pupils should be fluent in written methods for all four operations including long multiplication and division, and in working with fractions, decimals and percentages.

. + - + - + - + - + - +

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Children should be encouraged to estimate their answers before calculating.

Children should be encouraged to check their answers after calculation using an appropriate strategy.